# Statistical Catch-at-Length Assessment of S. fasciatus in Unit 3

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#### Abstract

An update of the 2011 Rademeyer and Butterworth SCAL assessment is presented. This incorporates some refinements of the previous methodology. Results with a deterministic stock-recruitment relationship are poor in not admitting a realistic estimate of survey catchability q. However, if the possibility of occasional large recruitments is introduced, the model fits the survey estimates of abundance better and a realistic estimate of q is obtained. As estimates of the depletion (B/K) of the resource vary considerably, possibly the best approach to management in the shorter term would be by setting catch limits based on annual replacement yield (RY) estimates, as these are reasonably robustly estimated at about 5000 tons.

## Introduction

This document presents results for an updated application of a Statistical Catch-at-Length (SCAL) assessment approach to the *S. fasciatus* resource in Unit 3. This Unit has the advantage, for assessment purposes, of minimal presence of *S. mentella*, and so provides a simple case for illustrating the SCAL methodology.

The results presented in this document fall into two sections. First there are those for some initial runs which were discussed at a teleconference held in early March 2014. Following that teleconference, ideas for further runs were offered and subsequently developed, and those follow in a second section.

## **Data and methods**

The data are as used in Rademeyer and Butterworth (2011) for *S. fasciatus* in Unit 3, and are reproduced in Appendix A.

The methodology, detailed in Appendix B, is also basically as described in Rademeyer and Butterworth (2011). The following changes have been made compared to that earlier paper.

- a. The growth parameters now used are:  $L_{inf}$ =31.879 (cm),  $\kappa$ =0.22132 (yr<sup>-1</sup>) and  $t_0$ =0 (from fitting a von Bertalanffy growth curve through the origin to the Campana ageing data from Units 1+2).
- b. Instead of assuming a knife-edged maturity-at-age 9, a knife-edged maturity-at-length 22 cm is assumed, which is then converted to maturity-at-age using the estimated age-length distribution.
- c. Although the survey biomass index is taken to be proportional to the mature biomass only (≥22cm), the model is now fitted to the whole range of survey catch-at-length data available (the assumption of proportionality to the mature biomass is carried over from simple models used in the past; it

might merit reconsideration when applying SCAL methodology which does not require this further specification).

- d. The survey and commercial catch-at-length data are downweighted by a factor of 0.01 instead of 0.1 in Rademeyer and Butterworth (2011). This is to ensure that catch-at-length information does not unduly influence the model's attempt to fit the survey index data.
- e. In the cases where log-normally distributed fluctuations about the stock-recruitment relationship are admitted, and with a high value for the extent of variability  $\sigma_R = 1.5$  to allow for the possibility of occasional very large recruitments, the starting abundance and age-structure corresponds to median rather than to mean recruitment (and carrying capacity K similarly), so that this reflects the typical situation *absent* those large year classes.
- f. The results for each run now include a value for replacement yield (RY). This is the future annual catch which would maintain the spawning biomass at its current (2010) level by 2020.

## Results

Results are first compared for a series of SCAL assessments with fixed q values (1.5, 1.0, 0.5 and 0.15) and first flat selectivity, followed by decreasing selectivity ("dome") at larger lengths (see below for the reasons why this approach of fixing to a series of fixed q values was adopted) (runs 1 to 8). Table 1 gives results for all these eight scenarios.

At the March 2014 teleconference, a further series of scenarios were suggested. The corresponding runs have been based on the q=0.5, flat selectivity at larger lengths, scenario.

- 9) Fixed q=0.43 (as advised to correspond to the estimate by Alida Bundy).
- 10) Estimate q freely.
- 11) Alternative growth curve see Figure 1 (Don Power, pers. commn).
- 12) Allow for recruitment variability with a)  $\sigma_R$ =0.4 and q=0.5, b)  $\sigma_R$ =1.5 and q=0.5 and c)  $\sigma_R$ =1.5 and q estimated freely.
- 13) Start the model in 1977 given lack of reliability of pre-1977 catches.
- 14) Allow for a change in commercial selectivity between 1986 and 1987.
- 15) a) Flat survey selectivity from length 25cm and b) flat survey and commercial selectivities from length 25cm.
- 16) a) A combination of 12b and 15b, and b) a combination of 12c and 15b, i.e. both high recruitment variability and flat selectivity.

Figures 2 to-8 compare the scenarios described above. These Figures contain plots of spawning biomass and recruitment (age-0 fish) trajectories (first row), fits to the survey and commercial catch-at-length data (second row, as averaged over all the years for which data are available) and fits to the survey biomass index, including residuals (third row).

Figures 2 and 3 compare scenarios across the different fixed q values for the flat selectivity (runs 1 to 4) and then the dome selectivity (runs 5 to 8) respectively. In these plots of the fits to the catch-at-length

data and the survey biomass index residuals, only the two extreme cases (q=1.5 and q=0.15) are shown. Figure 4 – 8 show results for the second set of scenarios, all compared to run 3 with q=0.5.. Figure 9 plots the commercial and survey selectivities-at-length for runs 3, 14 (change in commercial selectivity between 1986 and 1987), 15a (flat survey selectivity from length 25cm onwards) and 15b (flat commercial and survey selectivities from length 25cm onwards). The fit to the commercial CAL for run 3 and run 14 are compared in Figure 10.

In Appendix C, Figures C1.1 to AC.16b give results for each scenario individually. These Figures contain plots of spawning biomass, catch and recruitment trajectories as well as the stock-recruitment curve in the first row. Survey and commercial selectivities-at-length and -at-age are plotted in the second row, together with fits to the survey and commercial catch-at-length data (as averaged over all the years for which data are available). Bubble plots of the standardised residuals for the fit to the survey and commercial catch-at-length data (se proportional to the magnitude of the corresponding standardised residuals. For positive residuals the bubbles are grey, whereas for negative residuals the bubbles are white. Finally, the fit to the survey index, and the associated residuals, are plotted, together with the estimated distributions for length at age.

## Discussion

Initial discussion considers the first set of scenarios (runs 1-8), for which the stock-recruitment relationship is deterministic.

- 1) The survey biomass index data are too noisy to provide an unambiguous preferred fit. It iwas considered best initially to illustrate fits over a plausible range of values for *q*, which we has been taken to be 0.15 to 1.5 (note that values above 1 imply herding by the survey net). There will need to be further discussion as to what range IS reasonably considered plausible.
- 2) Over the range of q considered here, the resource is estimated to be above its  $B_{MSY}$  level in all the scenarios, and currently increasing. Estimates of current (2009) spawning biomass levels relative to pre-exploitation level range from 41 to 93% across the eight scenarios considered.
- 3) The priority is a good fit to the survey index. Although this index shows signs of first a downward then an upward trend, these models prefer a lower *q* with a fitted trend that is near flat. The reason is that the larger catches historically tend to have occurred BEFORE the survey index downtrend ends.
- 4) One MIGHT (no guarantee) get a better fit by trying out other values of M and h but we are skeptical that that will gain much, so wary about investing too much more time there.
- 5) The lower q fits better but we are nervous of over-interpreting that because this is achieved through a predicted index that is almost trendless, in contrast to apparent features in the survey data.
- 6) Introducing a selectivity dome does result in a better fit to the CAL data. Biomass and sustainable yield estimates increase, but the estimated status of the resource relative to K and to  $B_{MSY}$  is not greatly affected.

7) Fits to the CAL data might be improved through introducing recruitment and selectivity at age variability, plus smoothing the mean selectivity function with age.

Then for the further runs 5) to 16b) developed following the March 2014 teleconference, the following features are evident (see Table 2 and Figures 4-10).

- 8) Estimating *q* freely (run 10) leads to an unrealistically low value and correspondingly unrealistically high biomass.
- 9) A number of the sensitivity runs lead to little difference from the baseline run 3 (q = 0.5): the alternative growth curve (run 11); starting in 1977 (run 13), though biomass is less in this case; a change in commercial selectivity between 1986 and 1987 (run 14), which also does not improve the fit to the CAL data greatly (Figure 10); and forcing all selectivities to be flat above 25 cm (runs 15a and b).
- 10) With the introduction of stochasticity in recruitment, there is little difference to results if  $\sigma_R$  is small (run 12a). However for  $\sigma_R$  set large to allow for the possibility of occasional large year-classes (runs 12b and 12c), there is a distinct improvement to the fit to the survey abundance time series. MSY estimates for these scenarios are some 4-5 times larger than for the other scenarios considered.
- 11) Perhaps the best fits to these data are provided by the combination of large  $\sigma_R$  and flat selectivities above 25 cm (run 16a). This combination of assumptions also allows for a plausible estimate of q at 0.68 (run 16b) with a Hessian based CV of 0.68. Estimating rather than fixing q does not compromise estimation precision fatally: for example, the CV on the MSY estimate increases from 11 to 24%.

Finally, across all the scenarios considered (see also the plots in Appendix C) the following features are also evident.

- 12) Fits to the CAL data are not that good for the commercial catch, and improve only slightly for the surveys.
- 13) Estimates of replacement yield (RY) are certainly more robust than those of MSY. For most scenarios, these RY estimates range between 4300 and 5300 tons, thoughthey are slightly higher for the cases where *q* is fixed to be large (runs 1, 2 and 5).

## Conclusions

The most promising of the fits attempted are those which allow for the possibility of occasional high recruitments by setting the recruitment variability parameter  $\sigma_R$  large, though in future mixture distributions might offer a better way to model this possibility. They also admit a realistic estimate of catchability q, and without fatally jeopardising the precision of estimates.

Nevertheless estimates of the depletion (B/K) of the resource vary considerably. Possibly the best approach to management in the shorter term would be by setting catch limits based on annual replacement yield (RY) estimates, as these are reasonably robustly estimated at about 5000 tons

## REFERENCES

Rademeyer RA and Butterworth DS. 2011. Initial applications of statistical catch-at-age assessment methodology to Atlantic redfish. Document submitted to Canadian ZAP meeting related to Precautionary Approach reference points for redfish populations, Mont-Joli, October 2011: 34pp.

	1)	2)	3)	4)	5)	6)	7)	8)						
	Flat survey	and comm larger le	ercial select engths	ivities at	Decreasing survey and commercial selectivities at larger lengths									
	<i>q</i> =1.5	<i>q</i> =1.0	<i>q</i> =0.5	<i>q</i> =0.15	<i>q</i> =1.5	<i>q</i> =1.0	<i>q</i> =0.5	<i>q</i> =0.15						
-InL: overall	22.61	23.18	20.89	19.19	20.64	20.31	18.66	17.76						
-InL: survey	8.93	9.57	7.20	5.43	6.29	7.47	5.98	5.16						
-InL: survCAL	10.41	10.34	10.28	10.20	11.29	9.81	9.68	9.61						
-InL: comCAL	3.26	3.27	3.40	3.55	3.06	3.03	3.00	2.98						
-InL: RecRes	0	0	0	0	0	0	0	0						
h	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67						
М	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125						
θ	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00						
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00						
K <sup>sp</sup>	137	149	210	431	145	176	259	669						
B <sup>sp</sup> 2009	57	82	157	383	46	120	210	624						
B <sup>sp</sup> 2009/K <sup>sp</sup>	0.41	0.55	0.75	0.89	0.31	0.68	0.81	0.93						
MSYL <sup>sp</sup>	0.30	0.30	0.30	0.30	0.31	0.31	0.31	0.31						
B <sup>sp</sup> MSY	41	45	63	129	45	55	82	210						
MSY	6.7	7.1	9.6	19.5	6.6	8.1	11.7	29.9						
RY	6.4	5.8	4.7	4.4	6.8	5.3	4.7	4.5						
Survey	<i>q</i> 's	<i>q</i> 's	<i>q</i> 's	<i>q</i> 's	<i>q</i> 's	<i>q</i> 's	q's	<i>q</i> 's						
Unit 3	1.50	1.00	0.50	0.15	1.50	1.00	0.50	0.15						
$\sigma_R_{out}$	0	0	0	0	0	0	0	0						

**Table 1**: Results of fits of SCAL runs 1 to 8 for *S. fasciatus* in Unit 3. Values fixed on input rather than estimated are shown in **bold**. Mass units are '000t.

	5)	9)	10)	11)	12a)	12b)	12c)	13)	14)	15a)	15b)	16	a)	16	b)
	<i>q</i> =0.5	<i>q</i> =0.43	q estimated	Alt. growth curve	σ <sub>R</sub> =0.4, q=0.5	σ <sub>R</sub> =1.5, q=0.5	$\sigma_R$ =1.5, q estimated	Start in 1977	Change in sel in 1986	Flat survey sel >25cm	Flat survey and comm sel >25cm	Combin of 12b 15	nation o) and b)	Combin of 12c 15	nation :) and b)
-InL: overall	20.89	20.50	18.99	20.54	19.28	16.06	15.92	13.56	19.63	20.97	21.04	16.28		16.20	
-InL: survey	7.20	6.85	4.97	6.89	5.31	3.04	3.15	2.64	7.27	7.45	7.45	3.05		3.11	
-InL: survCAL	10.28	10.23	10.33	10.32	9.91	9.06	8.97	8.18	10.19	10.17	10.22	9.24		9.19	
-InL: comCAL	3.40	3.41	3.68	3.33	3.24	2.97	2.79	2.74	2.17	3.35	3.37	3.10		3.01	
-InL: RecRes	0	0	0	0	0.83	0.98	1.01	0	0	0	0	0.90		0.89	
h	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	-	0.67	-
М	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	-	0.125	-
θ	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.68	1.00	1.00	1.00	1.00	-	1.00	-
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-	0.00	-
K <sup>sp</sup>	210	222	4259	215	209	291	249	162	199	194	192	289	(0.11)	256	(0.26)
B <sup>sp</sup> 2009	157	171	4214	146	137	150	99	112	145	140	137	145	(0.27)	105	(0.78)
B <sup>sp</sup> 2009/K <sup>sp</sup>	0.75	0.77	0.99	0.68	0.65	0.52	0.40	0.69	0.73	0.72	0.72	0.50	(0.23)	0.41	(0.55)
MSYL <sup>sp</sup>	0.30	0.30	0.30	0.31	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	(0.06)	0.30	(0.06)
B <sup>sp</sup> <sub>MSY</sub>	63	67	1277	66	63	270	232	49	60	58	57	265	(0.14)	235	(0.26)
MSY	9.6	10.1	191.6	8.1	9.6	40.5	34.9	7.3	8.6	8.9	8.8	40.3	(0.11)	35.9	(0.24)
RY	4.7	4.7	5.0	5.1	5.3	4.9	4.5	4.3	4.6	4.9	4.9	4.5	-	4.6	-
Survey q's	0.50	0.43	0.01	0.50	0.50	0.50	0.78	0.50	0.50	0.50	0.50	0.50	-	0.68	(0.68)
$\sigma_R_{out}$	0	0	0	0	0.07	0.29	0.30	0	0	0	0	0.28	(0.60)	0.28	(0.62)

**Table 2**: Results of fits of SCAL runs 9 to 15 for *S. fasciatus* in Unit 3. Values fixed on input rather than estimated are shown in **bold**. Mass units are '000t. For runs 16a and 16b, the Hessian-based CVs are shown in parenthesis.



**Figure 1**: The base case growth curve used, as developed from ageing of *S. fasciatus* in Units 1+2 by Campana. An alternative growth curve (Don Power, pers. commn) used in run 11 is also shown.



**Figure 2**: Comparison of results for the four SCAL assessments of runs 1-4 with fixed q and **flat selectivity** at larger lengths. The fits to the survey and commercial CAL data (second row) are as averaged over all the years for which data are available.



**Figure 3**: Comparison of results for the four SCAL assessments with fixed *q* and **decreasing selectivity** at larger lengths (runs 5-8). The fits to the survey and commercial CAL data (second row) are as averaged over all the years for which data are available.



Figure 4: Comparison of results for runs 3 (*q*=0.5) and 9 (*q*=0.43), 10 (*q* estimated) and 11 (an alternative growth curve). The fits to the survey and commercial CAL data (second row) are as averaged over all the years for which data are available.



Figure 5: Comparison of results for runs 3 (q=0.5), 12a ( $\sigma_R$ =0.4, q=0.5), 12b ( $\sigma_R$ =1.5, q=0.5) and 12c ( $\sigma_R$ =1.5, q estimated). The fits to the survey and commercial CAL data (second row) are as averaged over all the years for which data are available.



**Figure 6**: Comparison of results for **runs 3** (*q*=0.5) and 13 (start in 1977). The fits to the survey and commercial CAL data (second row) are as averaged over all the years for which data are available.



Figure 7: Comparison of results for runs 3 (q=0.5), 14 (change in commercial selectivity between 1986 and 1987), 15a (flat survey selectivity from length 25cm) and 15b (flat survey and commercial selectivities from length 25cm). The fits to the survey and commercial CAL data (second row) are as averaged over all the years for which data are available.



Figure 8: Comparison of results for runs 3 (q=0.5), 16a ( $\sigma_R$ =1.5, q=0.5, and flat survey and commercial selectivities from length 25cm) and 16b ( $\sigma_R$ =1.5, q estimated, and flat survey and commercial selectivities from length 25cm). The fits to the survey and commercial CAL data (second row) are as averaged over all the years for which data are available.



Figure 9: Comparison of commercial and survey selectivities-at-lengths for runs 3, 14, 15a and 16b.

### Run 14: Change in comm. sel. between 1986 and 1987

Observed

Predicted

60

40

50

38

Run 8: *q*=0.5



Figure 10: Fit to the commercial CAL data for runs 3 and 14 (with change in commercial selectivity between 1986 and 1987).

## Appendix A - The data

Note: Units are throughout cm for length and yr for time.

**Table A1**: Catch in kt and swept area mature (i.e. >22cm) biomass estimates (in kt) and coefficients of variation (CVs) for *S. fasciatus* in management unit 3.

Year	Catch	Survey	CV
1960	20.10		
1961	19.60		
1962	24.00		
1963	23.50		
1964	10.80		
1965	11.00		
1966	25.90		
1967	6.60		
1968	2.90		
1969	5.40		
1970	15.70	55	(0.7)
1971	25.60	71	(0.7)
1972	24.40	133	(0.7)
1973	17.30	133	(0.7)
1974	14.20	31	(0.7)
1975	10.50	209	(0.7)
1976	7.00	26	(0.7)
1977	4.80	100	(0.7)
1978	3.70	169	(0.7)
1979	2.80	26	(0.7)
1980	4.00	15	(0.7)
1981	4.40	34	(0.7)
1982	4.70	71	(0.7)
1983	4.90	123	(0.7)
1984	5.20	96	(0.7)
1985	5.60	15	(0.7)
1986	6.60	79	(0.7)
1987	6.10	59	(0.7)
1988	3.90	79	(0.7)
1989	3.30	25	(0.7)
1990	2.30	56	(0.7)
1991	2.00	22	(0.7)
1992	2.50	107	(0.7)
1993	5.20	69	(0.7)
1994	5.20	47	(0.7)
1995	4.80	38	(0.7)
1996	4.80	42	(0.7)
1997	6.40	67	(0.7)
1998	5.80	17	(0.7)
1999	4.50	61	(0.7)
2000	4.80	48	(0.7)
2001	4.30	94	(0.7)
2002	4.80	32	(0.7)
2003	3.00	50	(0.7)
2004	2.10	33	(0.7)
2005	3.10	116	(0.7)
2006	2.70	96	(0.7)
2000	2.90	33	(0.7)
2008	3.60	146	(0.7)
2009	4,60	147	(0.7)
2010	5.20	- 17	(0.7)
2010	0.20		

Length	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
10-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	2	0	0	0	0	0	0	0	0	3	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	30	4	0	0	0	0	0	0	0	2	0	0	5	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39	19	57	10	0	0	0	0	0	0	0	5	21	5	0	0	0	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	9	50	78	24	0	0	2	4	0	3	0	12	30	11	11	5	0	24
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	27	0	0	61	0	18	111	146	49	10	15	9	2	0	19	2	14	69	22	12	9	3	88
17	0	0	18	144	0	0	0	0	0	0	0	0	0	0	0	10	11	18	2	23	0	245	0	63	314	197	74	13	27	3	14	0	36	0	20	134	97	42	33	42	190
18	0	25	0	96	0	0	0	0	0	0	0	0	2	0	0	13	62	6	0	75	33	294	0	69	501	261	97	72	147	51	61	0	117	47	20	235	260	91	74	138	777
19	24	0	87	776	0	0	17	0	0	15	8	31	18	7	5	26	150	135	85	72	68	453	0	304	565	381	173	176	204	151	277	1	270	98	51	176	259	249	291	543	2537
20	50	0	703	2147	191	41	17	87	0	46	23	86	104	23	9	114	232	221	89	244	71	563	0	379	660	655	275	654	303	519	705	3	814	304	205	166	241	374	504	1030	5198
21	386	39	1213	2278	667	53	94	211	25	60	35	165	117	53	35	123	387	663	73	478	165	1037	6	289	703	638	426	635	630	588	813	3	1229	523	354	244	233	377	754	1108	5508
22	549	151	2289	6714	2911	383	583	414	48	30	106	453	76	241	103	102	419	898	396	1014	216	508	19	874	942	775	696	1335	934	1144	1223	5	2061	1162	712	547	320	481	787	1054	4443
23	734	623	2286	7013	3716	1398	2106	690	112	147	123	560	163	228	161	248	473	1123	456	1202	534	575	19	696	1015	1071	868	1792	1182	1105	1367	5	1696	1065	601	873	478	656	1152	1081	2870
24	1011	1094	1749	6676	4582	2770	2357	1613	315	224	226	742	495	633	366	672	625	1387	530	1013	855	357	6	1295	1460	1256	1129	1984	1777	1641	1651	6	2400	1146	815	1156	524	772	1133	1185	1686
25	890	1705	1513	5927	4828	3499	3238	1233	475	576	363	815	994	956	767	1624	871	1897	768	1174	1176	418	16	1277	1634	1736	1771	1737	1673	1622	1584	6	2141	1263	1001	1183	660	809	1269	1156	1087
26	736	1699	1319	4768	4984	4121	2679	1661	750	838	435	1266	1430	1454	1266	1876	1331	2144	1077	1288	973	416	35	1115	1449	1842	2143	1891	1787	1578	1682	5	1845	1096	1015	1138	678	821	1072	1074	737
27	876	1883	1094	5328	6449	3540	2378	1619	812	803	733	950	1739	1575	1462	2263	1305	2027	1012	1110	1167	451	71	1119	1418	1646	2009	1544	1736	1285	1528	4	1413	933	727	1221	720	754	1002	1236	616
28	1182	2641	614	4038	3193	4357	1500	1282	534	867	644	1162	1305	1427	1722	1783	1201	1526	670	528	529	413	189	1270	1203	1363	1750	1318	1266	1075	990	3	959	520	560	1186	758	946	992	1138	538
29	1128	2764	682	3056	2520	2745	1143	972	590	1190	840	1143	985	1375	1103	1782	1038	1476	653	492	310	353	203	1298	1106	1209	1545	1199	1165	886	1002	2	776	443	444	872	633	710	944	1017	448
30	1258	2006	486	2650	2854	1940	987	855	620	873	783	1746	1000	1163	1229	1570	1140	1471	809	298	181	272	200	960	846	850	894	1106	1022	857	982	2	782	327	257	657	508	637	626	887	341
31	1425	2561	392	1927	1493	1707	1255	858	486	482	883	710	1078	953	1222	1116	869	953	396	403	226	168	190	678	498	463	447	556	594	424	464	1	424	195	134	298	463	531	455	693	315
32	1681	2457	538	1848	1299	1111	364	443	426	422	671	821	862	874	1119	882	752	842	555	326	242	113	241	638	467	448	319	528	533	295	397	1	291	172	125	169	356	426	416	532	371
33	1443	2620	511	1539	1350	1322	388	405	323	170	436	289	511	501	720	616	514	449	473	268	158	176	302	670	278	273	200	428	446	291	259	0	189	125	68	72	258	261	284	362	237
34	1835	3259	519	835	919	427	358	261	258	61	361	239	141	328	408	354	262	247	391	150	83	178	270	387	248	158	128	296	301	208	214	0	96	97	42	38	199	95	152	232	184
35	1732	2298	304	431	600	153	134	242	202	47	231	65	76	161	117	182	152	163	273	40	24	72	222	120	167	107	78	207	253	136	144	0	58	65	28	27	122	77	72	129	82
36	1351	2064	292	409	398	76	139	198	282	29	204	8	95	102	54	29	104	141	121	11	22	66	189	103	108	83	27	203	131	121	134	0	49	67	17	24	104	31	43	71	42
37	1050	1675	156	275	259	53	165	35	236	12	163	6	28	90	23	6	123	64	92	8	6	14	176	153	137	73	24	190	126	105	114	0	26	56	21	5	47	20	13	23	27
38	1090	1383	96	214	135	0	161	17	158	0	183	7	22	45	18	2	260	4	110	7	5	13	180	108	76	63	18	134	89	70	71	0	16	56	14	4	19	2	9	22	23
39	959	1208	65	40	110	0	93	0	141	1	93	4	5	16	10	2	169	9	109	3	2	0	285	79	47	39	10	88	80	67	65	0	12	44	8	4	18	5	7	19	9
40	898	1599	55	105	18	0	66	0	17	0	100	2	4	6	5	0	222	0	130	4	0	0	349	24	46	40	7	112	59	65	51	0	9	35	6	2	3	2	6	14	4
41	890	1512	77	0	18	0	36	0	145	0	34	0	1	2	2	0	143	0	67	1	0	0	163	0	35	13	3	60	31	38	31	0	7	22	5	1	0	0	1	8	2
42	806	1021	63	0	0	0	4	0	21	0	7	0	1	1	0	0	245	0	40	2	0	0	84	0	31	11	3	70	28	26	33	0	8	24	6	2	3	1	1	6	0
43	322	732	18	0	0	0	0	0	60	0	22	0	0	3	0	0	116	0	22	1	0	0	33	1	33	5	2	73	21	19	16	0	3	18	3	1	1	0	0	2	1
44	194	466	7	0	0	0	0	0	39	0	11	0	0	1	0	0	193	0	16	0	0	0	3	0	23	2	0	58	24	14	17	0	1	14	2	1	0	0	0	1	0
45	101	60	4	0	0	0	0	0	49	0	25	0	0	0	0	0	205	0	10	0	0	0	0	0	5	2	0	50	17	10	4	0	1	12	1	1	2	0	0	3	0
46	44	119	0	0	0	0	0	0	23	0	7	0	0	0	0	0	103	0	14	0	0	0	0	0	15	1	0	24	17	7	3	0	0	6	0	1	0	0	0	1	0
47	0	0	0	0	0	0	0	0	11	0	11	0	0	0	0	0	90	0	12	0	0	0	0	0	7	0	0	16	7	1	0	0	0	4	0	0	0	0	0	0	0
48	11	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	13	0	5	0	0	0	0	0	0	0	0	5	3	2	2	0	0	1	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	13	0	0	0	0	0	0	0	0	0	0	5	3	0	1	0	0	1	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table A2: Commercial catch-at-length (in thousands) for Atlantic redfish (assumed to be all S. fasciatus) for Unit 3 (Peter Comeau, pers. commn)

	Unit 3																																								
Length	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	175735	0	0	0	0	0	0	0	0	0	0	0	0	0	
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10474	0	0	0	0			0	28243	38303	0	0	0	0	0	0		0	19321	
4	0	ő	ő	ő	0	ő	ő	0	ő	ő	0	ő	0	ő	ő	6207	ő	ő	0	0	ő	46528	ő	0	ő	0	0	0	315289	208287	373424	0	39707	175361	0	156467	40388		548986	972636	137539
5	0	0	0	0	0	0	0	0	0	0	0	0	197323	13965	0	6207	6572	0	0	0	0	0	20236	9576	0	0	0	0	160035	204359	2029251	644007	76116	470751	407073	1193689	111795	19373	944654	1263906	533424
6	0	0	75505	0	0	0	0	0	0	0	0	0	466876	0	0	0	0	23001	0	174032	0	107412	20236	84415	44302	65122	225602	30330	143026	189313	326311	47257	172388	154055	1618922	238391	105349	38746	165930	51811	487392
7	0	0	81712	0	0	0	0	0	11518	0	0	47538	877224	7199	128932	0	68332	148637	49898	279094	110350	50067	17164	319684	151071	46750	387127	44011	767397	161469	17239	104401	1095154	1500796	1061478	161350	199815	126336	217390	547327	721976
8	0	9598	196928	39592	6062	0	0	0	48743	0	0	0	661806	169767	114800	29824	196581	616668	157179	158215	591329	1058007	6745	1371337	428579	177062	463133	213366	1619585	267643	97285	104401	3506090	2150781	2155824	1928727	524331	587636	105572	1276710	1665323
9	0	0	223842	19196	11526	0	39182	0	60244	6775	0	61741	599687	339635	0	89473	309595	1498602	597402	445544	1810130	905499	93068	2554752	793665	315407	1030674	216842	966810	1305871	247999	227885	3152960	908498	4649888	5507535	814593	293816	193692	1462262	2058851
10	257194	/3830	410204	78893	170457	0	22129	18827	48743	75369	42636	48317	427834	2292671	320900	24712	041519	1408661	1159672	1182506	1305759	3744032	303381	1520271	2420664	352921	030844	450861	247760	1271534	140654	20194	2727926	45179629	7311360	8204953	2019646	220459	228390	729831	1944523
12	70482	459192	406080	489867	332041	7980	0	160885	20490	27312	42636	0	338580	2071165	1181626	312472	758952	131973	5740587	2961668	745376	11617931	285477	1552809	9140692	894603	5297602	422263	684245	2114598	1146540	411529	897651	81618891	4722885	3017645	6006680	628402	289843	1101060	1219591
13	19196	253735	372533	1236338	1838556	54297	53050	194836	131706	14005	19475	0	234090	606977	2859043	504737	554830	515013	6056755	3129740	1174802	1708445	192340	1478006	9678941	1402691	5198744	844546	1827293	2641555	1944967	711385	1063189	20615978	3119036	6940186	3272361	391456	580367	4399730	2733769
14	358996	1150409	541517	1245130	5552519	166963	0	82790	175094	54313	32235	123172	192621	803997	6713216	1590811	556675	206648	1692772	3291009	1445525	1792579	73471	2083994	9926319	1621339	2690677	1304259	3131677	2436125	2702222	866251	606537	8592425	5230786	10120026	4682137	2702176	2965078	2657581	2245253
15	1840797	1487561	314910	234900	14930508	207373	5759	165187	488732	78601	23158	76487	225710	453609	10789260	2584524	581986	314619	782531	2201116	1017089	1809554	400540	1825710	8491664	2531356	2236894	4438932	4283633	1261886	3636908	965918	469336	2926878	7218102	9957873	10267842	2835603	1291433	4748740	2794656
16	3458498	2210358	555258	440371	22357270	378361	250586	38805	1198477	370196	36254	641122	622773	162315	12125780	3004433	1099283	287251	697327	2609353	1837410	1642826	417555	1701336	6806156	1886927	3714263	8525382	3295585	1551955	6306215	1763267	633315	2345082	5411008	7987711	24632110	5786391	4200634	10970013	2740556
17	9466424	11366836	2059214	170869	24303134	320587	939992	106732	353697	534050	156638	436298	684464	508583	7846761	3354144	1723078	420048	424193	2792221	4910129	1462930	224025	1576179	5585691	2665519	6863454	17067535	5615412	3354350	4974305	1337732	1256370	2368609	3397136	8239690	62300526	9157632	35551576	18744808	2827757
18	20242115	30057765	6371871	1050130	12642409	735776	1424167	183966	152488	546391	0	823940	2666558	438352	4447717	3147922	3160778	539163	370129	2159350	6274797	2163785	1340894	2303595	4309473	6634483	6946490	31773744	13173393	4679471	5792658	1569713	2139154	4643623	3171650	10619594	90664421	37943995	77589246	70847955	5469987
19	30426886	27038532	13993192	3865163	4286/30	622955	4895591	1397990	567564	1045406	1255930	291018	2206538	009832	1823832	2209770	4261185	1026939	1221502	181/224	8243895	2626922	13/5452	212/48/	2/983/2	9683304	11325/56	94297750	1327/965	10486463	12247788	19/96/6	2145546	4348339	2423774	3733701	/1181/33	00579885	153301280	22/219691	10535189
20	53586281	38333456	34158702	12392050	2165382	3258812	7148094	5506015	82851	885122	1878104	1149446	5616079	1044718	2555367	1424575	10609745	3377375	3020525	1338074	9110181	3780088	1638567	4418553	3310622	11986228	10733700	82258586	5073312	25517220	26320747	6717354	6991563	10321446	7493509	8875305	26702124	61948073	203473735	419608591	564374403
21	53366300	41483809	41124477	39644145	7335126	5394013	7364934	15694629	941824	736182	2659888	1933445	6691944	2584793	2069312	675569	10960235	4444456	8216726	1611686	10101261	4946298	1372426	6430519	4750621	11436313	11904232	77788169	10155560	35455589	28323398	30675733	14779226	18812947	14641030	9839779	27734084	42397819	169516871	288469437	63229362
23	41725039	41290995	57367507	76648499	7068714	18238760	5113079	26220958	5141850	510766	3517169	4116575	16495643	5407636	4467021	404688	7116835	3951493	12139852	3498409	15809292	5958652	18441633	9844560	7638971	11515623	17698921	51407840	6297256	40483094	21537987	35960656	18932307	26019744	22607602	15857252	24818632	30559363	107737979	205189420	54582264
24	30417436	36836853	71077862	76892816	6850946	43298620	2994483	37059700	11888381	1250817	5780905	4452511	21716784	15326495	4762010	379249	9512944	4881436	16762173	4345079	26268964	5934922	28280597	15160671	12144259	14327677	16713591	54867465	7992028	35048139	19194450 4	46796738	19714610	33207724	16834310	28340025	23552296	21728043	87782451	122826891	45816873
25	23035707	38929644	54802378	82260655	6554041	63702354	1443428	32283240	26549466	1740157	6370128	6469720	25444594	25762846	12928106	728216	8206110	7261950	25881702	3812315	30858304	5817910	50057966	19554333	19803361	16242417	16086719	23752740	7590232	33314995	15026139	50691626	14728014	30601972	15893392	40297603	29630868	15335372	48552651	61777636	36058670
26	17558337	31423698	44436318	72226991	5956873	82346597	1604372	33057808	56862786	2664938	8432509	4902336	23721928	53561021	11934796	1006539	14217565	14484422	27814983	4969723	22705642	5751515	53776698	19728643	22820073	10239183	16451771	14519259	9425007	25979242	17900061	49007789	12638175	29086478	14523463	47654026	29873073	14225537	46754795	39590602	20016174
27	14590071	24953543	29024455	53684728	6686691	76890739	2401130	25997411	65849876	3838860	4341821	7598910	16044489	60667826	21060610	2041975	14392916	18363050	27799214	3057830	22095174	4959035	36149871	17182292	17097903	9006937	16043120	17227526	5611637	22297425	17619903	33272794	9602434	17548761	12732694	44668085	31746964	9300715	35033107	29854667	19148775
20	8833580	10477772	22557405	19870707	5740060	63165452	4090631	29432028	63297276	3307232	3743261	10067277	10273373	25453325	15831078	2921000	10703018	14769411	18571070	3359607	6953130	3152842	74049409	15696970	10050284	0584264	11570489	11300568	2964806	14436679	15546883	19801436	5573479	8311845	5600313	45057907	20042490	7338494	33177517	2003/40/	0187416
30	9014271	11998185	22359459	14236849	4242303	37777599	5267318	21544319	47241913	3104535	1757999	8092368	15094202	24890964	13396189	2500646	11162321	12970317	15480421	3192779	7363192	2452184	17646989	13319320	9907047	5873549	9208769	11181303	1743249	7916152	13097567	16035731	3860718	4949036	4284584	28825783	18164845	4087784	32349877	16476355	4450692
31	6198025	6469515	20784491	9150943	5628910	45333966	4360721	11790419	29026847	3229022	1192439	5435913	11513736	19783592	13977498	2151224	8482896	14051859	15278312	3929162	6144965	3525310	11156678	8354031	5023528	3787479	3335358	8351481	1111303	5843865	7468928	11023024	2319843	2097781	3076302	22165929	17363104	3529854	22473510	10511412	7654495
32	4424703	4195342	10287982	8451616	4635130	5088147	5005238	9504517	16575230	2677926	649440	3414056	16436788	21196956	17318867	2380770	8728459	16474488	11100440	3196248	3986397	2752016	8105519	9109475	3628279	4934472	4649122	6692940	1655796	3188187	3862897	4709318	1424579	2384981	2482390	11310164	10938428	1574835	12588200	7996018	4765627
33	2039105	3914222	11079466	5820353	3605729	15644529	5140953	5164039	17786069	5338364	559954	1750545	4346390	10681660	14379856	2990679	5006873	6492403	7096381	2509323	2429614	1879513	10952874	7306540	2901758	3357833	1553420	5365998	1031868	2016822	4029108	3562456	645475	1569183	1298448	8008262	13038802	1122576	8974513	4707020	3197477
34	2497046	3491061	8148458	2908767	2800414	10643823	3360600	2022475	7287158	3999766	424183	1625496	3275377	6082529	10444752	1062441	5574765	3740167	4194384	1832244	2518501	726942	6049760	6376592	1393890	2506339	854242	2305494	181170	1821514	456546	3261840	1206020	1800512	1620875	3349431	7022283	771582	6019344	2489861	2845139
35	2069079	2534130	/390668	1/41160	1358378	3603776	1864943	2700669	2591623	2732365	420360	1539740	1165414	4351960	4/02/14	1111847	2657658	2/16218	3807714	1609267	962205	588016	3023875	2887780	/85535	2141/8/	341642	2052571	1/1252	/61/46	295152	1739818	680131	133990	808345	1210778	3452/11	403137	2547994	/86135	1568256
35	108/653	1152474	11094075	2508189	2033047	1231170	1591492	1536432	1609264	1492242	148357	1093436	12/9065	1086053	4214677	1433070	1908935	958220	2056514	1080574	1099958	374058	163/11/	2431243	816122	454113	53918/	207525	94/6/	613095	403118	873460	632873	139289	498477	126/214	747000	333195	647840 31E100	261647	480498
38	562620	426900	3204984	832929	1436779	3094801	1016752	883025	909279	2105234	151196	1115236	904575	1120183	3617271	1579993	4724217	367050	1030823	1211358	274449	506365	709736	1180156	592991	989406	2913/9	210667	139752	258252	262356	165342	625158	65719	154198	177069	107550	131243	289529	79537	200709
39	269964	242986	1820191	343274	1099072	2800777	915552	882714	566786	1407114	63475	1048296	248953	1181689	2424191	643652	7102244	126929	395220	957116	337969	698528	594798	867259	180971	342677	ő	265850	74729	71034	81813	177279	696237	69206	128938	23069	114542	131242	0	0	00000
40	302357	228131	349394	297462	438038	2102049	683151	639237	611578	571127	99674	946334	482491	752128	1626038	407154	6920072	111915	361156	841906	121588	503821	654311	616673	291836	45604	29991	179747	130046	149481	22161	22161	217025	122221	86779	123127	47691	78838	0	0	c
41	181896	138065	0	61543	249331	995992	24827	348366	83755	202263	72882	491551	265616	493320	1246474	110844	5162816	76151	183962	378798	242302	187026	21915	0	113354	43446	0	87264	195068	22541	110118	137876	81054	46137	41660	23069	0	98008	22413	0	C
42	214290	168960	0	15436	533137	567045	25735	255062	168993	32193	63475	362560	67786	336293	80140	45280	2504389	31732	45572	427968	39628	150226	84090	328355	113354	45604	0	0	109901	28862	44832	0	82205	115343	125893	0	0	32618	0	0	131180
43	118777	39447	0	58903	339601	116346	64044	180277	18867	64208	0	214531	75132	78523	124309	22640	1076656	24654	46408	277668	47871	217561	81899	0	68012	0	0	0	131576	0	22161	0	69247	46137	33077	23069	0	39419	22038	0	80159
44	64/8/	19724	0	0	80857	450339	6760	144640	/8894	0	0	185231	45079	27660	24654	6/195	239257	26842	0	31/53	0	32078	124350	269452	45342	22802	50205	0	43349	29824	226/1	22161	22413	50570	21915	69206	0	0	0	61920	44076
45	43192	19/24	0	0	6300	360190	3739	00513	0	0	0	10909	15076	13830	0	0	0		0	63506	0	87000	0	0	0	0			23204	22341	0	22101	21550	23069	19745	25009	0		0	0	22030
47	0	0	0	0	0	360190	0	ň	0	ő	0	0	1502.0	15050	0	0	0	0	ň	00000	0	0	0	ň	0	0			0	0	20453	ő	0	0	9872	0	0		0	ň	
48	0	0	0	0	0	270143	0	0	0	0	0	0	0	0	122605	0	0	55958	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C
49	0	0	0	0	0	450238	0	0	0	0	0	0	0	0	0	0	0	0	0	32193	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C
50	0	0	0	0	0	270143	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C
51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0	. 0	0	0	
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	. 0		0	0	0	0	0	0	0	0	0	. 0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	ő	0	0	0	ő	0	0	0	ő	0	0	0	ő	0	0	0	0		0	0	0	0	0	ő	0	0	0	. 0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	c

Table A3: Survey catch-at-length (numbers) for S. fasciatus for Unit 3 (Peter Comeau, pers. commn)

S. fasciatus				
М	0.125			McAllister and Duplisea (2012)
h	0.67			McAllister and Duplisea (2012)
Length-at-maturity	22			Knife-edged, Don Power, pers. commn
Fraction of $M$ that occurs before spawning ( $M^{s}$ )	0.25			
Length-at-age	L <sub>inf</sub> 31.88 α	κ 0.2213 β	t <sub>0</sub> 0	$L_a = L_{ m inf} \Bigl( {f l} - e^{-\kappa (a - t_0)} \Bigr)$ , Campana, pers. commn
Weight-at-age	0.01106	3.08		$W_a = lpha \left( L_a  ight)^{eta}$ , McAllister and Duplisea (2012)

 Table A4: Life history parameters assumed for S. fasciatus.

## Appendix B - The Statistical Catch-At-Length Model

The model used for these assessments is a Statistical Catch-At-Length (SCAL) model. The approach used involves the construction of an age-structured model of the population dynamics and fitting it to the available abundance indices by maximising the likelihood function. The general specifications of the model and its equations are described below, followed by details of the contributions to the (penalised) log-likelihood function from the different sources of data available and assumptions concerning the stock-recruitment relationship. Quasi-Newton minimization is used to minimize the total negative log-likelihood function (the package AD Model Builder<sup>™</sup>, Otter Research, Ltd is used for this purpose).

#### **B.1.** Population dynamics

#### B.1.1 Numbers-at-age

The resource dynamics are modelled by the following set of population dynamics equations:

$$N_{y+1,0} = R_{y+1}$$
(B1)

$$N_{y+1,a+1} = \left(N_{y,a} e^{-M_a/2} - C_{y,a}\right) e^{-M_a/2} \qquad \text{for } 0 \le a \le m-2$$
(B2)

$$N_{y+1,m} = \left(N_{y,m-1} e^{-M_{m-1}/2} - C_{y,m-1}\right) e^{-M_{m-1}/2} + \left(N_{y,m} e^{-M_m/2} - C_{y,m}\right) e^{-M_m/2}$$
(B3)

where

 $N_{v.a}$  is the number of fish of age *a* at the start of year *y* (which refers to a calendar year),

 $R_y$  is the recruitment (number of 0-year-old fish) at the start of year y,

 $M_a$  denotes the natural mortality rate for fish of age a,

 $C_{y,a}$  is the predicted number of fish of age *a* caught in year *y*, and

*m* is the maximum age considered (taken to be a plus-group), *m*=20.

These equations reflect Pope's form of the catch equation (Pope, 1972) (the catches are assumed to be taken as a pulse in the middle of the year) rather than the more customary Baranov form (Baranov, 1918) (for which catches are incorporated under the assumption of steady continuous fishing mortality). Pope's form has been used in order to simplify computations. As long as mortality rates are not too high, the differences between the Baranov and Pope formulations will be minimal.

#### B.1.2. Recruitment

The number of recruits at the start of year y is assumed to be related to the spawning stock size (i.e. the biomass of mature fish) by a Beverton-Holt stock-recruitment relationship (Beverton and Holt, 1957), parameterised in terms of the "steepness" of the stock-recruitment relationship, h, and the pre-

exploitation equilibrium spawning biomass,  $K^{sp}$ , and recruitment,  $R_0$  and allowing for annual fluctuation about the deterministic relationship:

$$R_{y} = \frac{4hR_{0}B_{y}^{sp}}{K^{sp}(1-h) + (5h-1)B_{y}^{sp}}e^{(\varsigma_{y} - \sigma_{R}^{2}/2)}$$
(B4)

where

- $\varsigma_y$  reflects fluctuation about the expected recruitment for year y, which is assumed to be normally distributed with standard deviation  $\sigma_R$  (which is input in the applications considered here); these residuals are treated as estimable parameters in the model fitting process.
- $B_{y}^{sp}$  is the spawning biomass at the start of year y, computed as:

$$B_{y}^{sp} = \sum_{a=1}^{m} f_{a} w_{a}^{strt} N_{y,a} e^{-M_{a}M^{s}}$$
(B5)

where

 $w_a^{strt}$  is the mass of fish of age *a* during spawning,

- $f_a$  is the proportion of fish of age *a* that are mature
- $M^{s}$  is the fraction of mortality that occurs before spawning ( $M^{s} = 0.25$ ).

In the fitting procedure,  $K^{sp}$  is estimated while *h* has thus far been fixed at 0.67 for consistency with McAllister and Duplisea (2011).

#### B.1.3. Total catch and catches-at-age

The catch-at-age in year y is given by:

$$C_{y,a} = N_{y,a} \ e^{-M_a/2} S_{y,a} F_y \tag{B6}$$

where

 $S_{y,a}$  is the commercial selectivity (i.e. combination of availability and vulnerability to fishing gear) at age *a* and in year *y*; when  $S_{y,a} = 1$ , the age-class *a* is said to be fully selected, and

 $F_{v}$  is the proportion of a fully selected age class that is fished.

Selectivity is estimated as a function of length and then converted to selectivity-at-age:

$$S_{y,a} = \sum_{l} S_{y,l} A_{a,l} \tag{B7}$$

where  $A_{a,l}$  is the proportion of fish of age *a* that fall in the length group *l* (i.e.,  $\sum_{l} A_{a,l} = 1$  for all ages).

The matrix  $A_{a,l}$  is calculated under the assumption that length-at-age is normally distributed about a mean given by the von Bertalanffy equation, i.e.:

$$L_a \sim N[L_{\infty}(1 - e^{-\kappa(a - t_o)}); \theta_a^2]$$
(B8)

where

 $\theta_a$  is the standard deviation of length-at-age *a*, which is taken as proportional to the expected lengthat-age *a*, i.e.:

$$\theta_a = \beta^* L_{\infty} \left( 1 - e^{-\kappa (a - t_o)} \right) \tag{B9}$$

with  $\beta^*$  an estimable parameter.

The model estimate of the survey biomass is calculated as:

$$B_{y}^{surv,i} = \sum_{a=1}^{m} \widetilde{w}_{y,a}^{mid} S_{a}^{surv,i} N_{y,a} e^{-M_{a} \frac{m^{surv,i}}{12}} \left( 1 - S_{a} F_{y} \frac{m^{surv,i}}{12} \right)$$
(B10)

where

 $S_a^{surv,i}$  is the survey selectivity for age *a* for survey *i*,

 $m^{surv,i}$  is the month in which survey takes place (  $m^{surv,i} = 7$  ), and

 $\widetilde{w}_{y,a}^{mid}$  is the selectivity-weighted mid-year weight-at-age a landed in year y, and

$$\widetilde{w}_{y,a}^{\text{mid}} = \sum_{l} S_{y,l} w_{l} A_{a,l} / \sum_{l} S_{y,l} A_{a,l}$$
(B11)

with

 $w_l$  being the weight of fish of length *l*.

#### B.1.4. Initial conditions

For the first year  $(y_0)$  considered in the model therefore, the stock is assumed to be at a fraction ( $\theta$ ) of its pre-exploitation biomass, i.e.:

$$B_{y_0}^{sp} = \theta \cdot K^{sp} \tag{B12}$$

with the starting age structure:

$$N_{y_0,a} = R_{start} N_{start,a} \qquad \qquad \text{for } 0 \le a \le m \tag{B13}$$

where

$$N_{start,0} = 1 \tag{B14}$$

$$N_{start,a} = N_{start,a-1} e^{-M_{a-1}} (1 - \phi S_{a-1}) \qquad \text{for } 1 \le a \le m - 1 \tag{B15}$$

$$N_{start,m} = N_{start,m-1} e^{-M_{m-1}} (1 - \phi S_{m-1}) / (1 - e^{-M_m} (1 - \phi S_m))$$
(B16)

where  $\phi$  characterises the average fishing proportion over the years immediately preceding  $y_0$ .

Unless indicated otherwise though, the stock is assumed to be at pristine equilibrium in 1960, i.e.  $\theta = 1$  and  $\phi = 0$  for the results reported here.

#### B.2. The (penalised) likelihood function

The model can be fit to survey abundance indices, and commercial and survey catch-at-length data to estimate model parameters (which may include residuals about the stock-recruitment function, the fishing selectivities, the annual catches or natural mortality, facilitated through the incorporation of penalty functions described below). Contributions by each of these to the negative of the (penalised) log-likelihood (- $\ell nL$ ) are as follows.

#### B.2.1. Survey abundance data

The likelihood is calculated assuming that the observed survey index is log-normally distributed about its expected value:

$$I_{y}^{i} = \hat{I}_{y}^{i} \exp\left(\varepsilon_{y}^{i}\right) \quad \text{or} \quad \varepsilon_{y}^{i} = \ln\left(I_{y}^{i}\right) - \ln\left(\hat{I}_{y}^{i}\right) \tag{B17}$$

where

 $I_{v}^{i}$  is the survey biomass index for year y and survey i,

 $\hat{I}_{y}^{i} = \hat{q}^{i} \hat{B}_{y}^{surv,i}$  is the corresponding model estimate, where  $\hat{B}_{y}^{surv,i}$  is the model estimate of survey biomass, given by equation (B10),

 $\hat{q}^i$  is the constant of proportionality (catchability) for survey series *i*, and

$$\varepsilon_y^i$$
 from  $N\left(0, \left(\sigma_y^i\right)^2\right)$ .

The contribution of the survey biomass data to the negative of the log-likelihood function (after removal of constants) is then given by:

$$-\ln L^{surv} = \sum_{i} \sum_{y} \left[ \ln \left( \sigma_{y}^{i} \right) + \left( \varepsilon_{y}^{i} \right)^{2} / 2 \left( \sigma_{y}^{i} \right)^{2} \right]$$
(B18)

where

 $\sigma_{y}^{i}$  is the standard deviation of the residuals for the logarithm of survey index *i* in year *y*.

The catchability coefficient  $q^i$  for survey index *i* is estimated by its maximum likelihood value:

$$\ell n \, \hat{q}^{i} = 1/n_{i} \sum_{y} \left( \ln I_{y}^{i} - \ln \hat{B}_{y}^{surv,i} \right) \tag{B19}$$

#### B.2.2. Commercial catches-at-length

The contribution of the catch-at-length data to the negative of the log-likelihood function under the assumption of an "adjusted" (or "Punt-Kennedy (1997)") lognormal error distribution is given by:

$$- \ln L^{CAL} = W_{CAL} \sum_{y} \sum_{l} \left[ \ln \left( \sigma_{com} / \sqrt{p_{y,l}} \right) + p_{y,l} \left( \ln p_{y,l} - \ln \hat{p}_{y,l} \right)^2 / 2 \left( \sigma_{com} \right)^2 \right]$$
(B20)

where

 $p_{y,l} = C_{y,l} / \sum_{l'} C_{y,l'}$  is the observed proportion of fish caught in year y that are of length *l*,  $\hat{p}_{y,l} = \hat{C}_{y,l} / \sum_{l'} \hat{C}_{y,l'}$  is the model-predicted proportion of fish caught in year y that are of length *l*,

where

$$\hat{C}_{y,l} = N_{y,a} A_{a,l} S_{y,lo} e^{-M_a/2} F_y$$
(B21)

and  $\sigma_{com}$  is the standard deviation associated with the catch-at-length data, which is estimated in the fitting procedure by:

$$\hat{\sigma}_{com} = \sqrt{\sum_{y} \sum_{l} p_{y,l} (\ln p_{y,l} - \ln \hat{p}_{y,l})^2 / \sum_{y} \sum_{l} 1}$$
(B22)

The log-normal error distribution underlying equation (B20) is chosen on the grounds that (assuming no ageing error) variability is likely dominated by a combination of interannual variation in the distribution of fishing effort, and fluctuations (partly as a consequence of such variations) in selectivity-at-age, which suggests that the assumption of a constant coefficient of variation is appropriate. However, for ages poorly represented in the sample, sampling variability considerations must at some stage start to dominate the variance. To take this into account in a simple manner, motivated by binomial distribution properties, the observed proportions are used for weighting so that undue importance is not attached to data based upon a few samples only.

The  $W_{CAL}$  weighting factor is set to 0.01 to downweight the contribution of the catch-at-length data (which tend to be positively correlated between adjacent length groups) to the overall negative log-likelihood compared to that of the survey biomass data.

Commercial catches-at-length are incorporated in the likelihood function using equation (B20), for which the summation over age *I* is taken from length  $I_{minus}$  (considered as a minus group) to  $I_{plus}$  (a plus group), see Table B1.

#### B.2.3. Survey catches-at-length

The survey catches-at-length are incorporated into the negative of the log-likelihood in an analogous manner to the commercial catches-at-length, assuming an adjusted log-normal error distribution (equation (B20)) where:

 $p_{y,l}^{i} = C_{y,l}^{surv,i} / \sum_{l'} C_{y,l'}^{surv,i}$  is the observed proportion of fish of length *l* in year *y* for survey series *i*,

 $\hat{p}_{y,l}^{i}$  is the expected proportion of fish of length *l* in year *y* in the survey *i*, given by:

 $\hat{p}_{y,l}^{i} = \hat{C}_{y,l}^{i} / \sum_{l'} \hat{C}_{y,l'}^{i}$  is the model-predicted proportion of fish caught in year y and survey i that are of length *l*,

where

$$\hat{C}_{y,l}^{i} = N_{y,a} A_{a,l} S_{l}^{surv,i} e^{-M_{a} \frac{m^{surv,i}}{12}} (1 - S_{a} F_{y} \frac{m^{surv,i}}{12})$$
(B23)

Survey catches-at-length are incorporated in the likelihood function using equation (B20), for which the summation over age *I* is taken from length  $I_{minus}$  (considered as a minus group) to  $I_{plus}$  (a plus group), see Table B1.

#### B.2.4. Stock-recruitment function residuals

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The stock-recruitment residuals are assumed to be log-normally distributed. Thus, the contribution of the recruitment residuals to the negative of the (now penalised) log-likelihood function is given by:

$$-\ell n L^{SRpen} = \sum_{y=y1}^{y2} \left[ \varepsilon_y^2 / 2\sigma_R^2 \right]$$
(B24)

where

$$\varepsilon_y$$
 from  $N(0, (\sigma_R)^2)$ , which is estimated for year y1 to y2 (see equation (B4)), and

 $\sigma_R$  is the standard deviation of the log-residuals, which is input ( $\sigma_R = 0.4$  or  $\sigma_R = 1.5$ )

Table B1: Minus and plus length groups (in cm) for the commercial and survey CAL. Note:  $I_{min}$  for the surveys is not taken as a minus group.

	S. fasciatus
Commercial CAL:	
I <sub>minus</sub>	20
I plus	40
Survey CAL:	
I <sub>minus</sub>	10
I plus	40

#### **B.3. Model parameters**

#### B.4.1. Fishing selectivity-at-length:

The commercial and survey fishing selectivity-at-length,  $S_l$  and  $S_l^{surv,i}$  are estimated directly for a series of lengths (see Table B2) and is taken to be linear between these lengths. The slope from lengths  $I_{minus}$  to  $I_{minus}$ +1 is assumed to continue exponentially to lower lengths down to length 1. For lengths above  $I_{plus}$ , the selectivity is taken either to be flat (i.e.  $S_l = S_{I_{plus}}$  for  $l > I_{plus}$ ) or decreasing exponentially (i.e.  $S_l = S_{I_{plus}} e^s$  for  $l > I_{plus}$ , with s an estimable parameter).

The selectivities-at-length are then converted to an effective selectivity at age  $\widetilde{S}_a$ :

$$\widetilde{S}_{a} = \widetilde{w}_{a}^{mid} / w_{a}^{mid}$$
(B25)

with

$$\widetilde{w}_{a}^{mid} = \sum_{l} S_{l} w_{l} A_{a+1/2,l}$$
(B26)

 $\widetilde{w}_{a}^{\textit{mid}}$  is the selectivity-weighted mid-year weight-at-age a , and

 $w_l$  is the weight of fish of length *l* 

Table B2: Lengths (cm) at which commercial and survey selectivity is estimated directly.

Commercial CAL:	20	25	30	35	40		
Survey CAL:	10	15	20	25	30	35	40

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# Appendix C: Full set of results for runs 1 to 16b

Figure C.1: Full set of results for the SCAL assessment with *q*=1.5 and flat selectivity at larger lengths (run 1).



Figure C.2: Full set of results for the SCAL assessment with *q*=1.0 and flat selectivity at larger lengths (run 2).



Figure C.3: Full set of results for the SCAL assessment with *q*=0.5 and flat selectivity at larger lengths (run 3).



Figure C.4: Full set of results for the SCAL assessment with *q*=0.15 and flat selectivity at larger lengths (run 4).



Figure C.5: Full set of results for the SCAL assessment with *q*=1.5 and decreasing selectivity at larger lengths (run 5).



Figure C.6: Full set of results for the SCAL assessment with *q*=1.0 and decreasing selectivity at larger lengths (run 6).



Figure C.7: Full set of results for the SCAL assessment with *q*=0.5 and decreasing selectivity at larger lengths (run 7).



Figure C.8: Full set of results for the SCAL assessment with *q*=0.5 and decreasing selectivity at larger lengths (run 8).



Figure C.9: Full set of results for the SCAL assessment with q=0.43 (run 9).



Figure C.10: Full set of results for the SCAL assessment with *q* estimated (run 10).



Figure C.11: Full set of results for the SCAL assessment with alternative growth curve (run 11).



**Figure C.12a**: Full set of results for the SCAL assessment with q=0.5 and  $\sigma_R=0.4$  (run 12a).



Figure C.12b: Full set of results for the SCAL assessment with q=0.5 and  $\sigma_R=1.5$  (run 12b).



Figure C.12c: Full set of results for the SCAL assessment with q estimated and  $\sigma_R$ =1.5 (run 12c).



Figure C.13: Full set of results for the SCAL assessment with a start in 1977 (run 13).



Figure C.14: Full set of results for the SCAL assessment with a start in 1977 (run 14).



Figure C.15a: Full set of results for the SCAL assessment with flat survey selectivity from 25cm onwards (run 15a).



Figure C.15b: Full set of results for the SCAL assessment with flat survey and commercial selectivities from 25cm onwards (run 15b).



**Figure C.16a**: Full set of results for the SCAL assessment with q=0.5,  $\sigma_R=1.5$  and flat survey and commercial selectivities (run 16a).



Figure C.16b: Full set of results for the SCAL assessment with q estimated ,  $\sigma_R$ =1.5 and flat survey and commercial selectivities (run 16b).